

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 4, May 27, 2019¹

1. (a) What is the remainder when 2^{2019} is divided by 7?
(b) Find the last digit of 2^{2019} .
2. Gerald rolls 5 dice simultaneously. Each die has six faces labelled with the numbers 1;2;3;4;5 and 6. What is the probability of getting 5 consecutive numbers as the outcome of the dice roll?
3. If $y = 2$ and

$$\begin{array}{ccccccc} s & \text{---} & r & \text{---} & q & \text{---} & p & \text{---} \\ x + & & y + & & x + & & y + & \dots = 7; \end{array}$$

Senior Questions

1. Consider the points of intersection of the graphs $y = \cos x$ and $x = 100 \cos(100y)$ for which both coordinates are positive. Let a be the sum of their x -coordinates and b be the sum of their y -coordinates. Determine the value of $\frac{a}{b}$.
2. Prove that $\log_a(x) \log_b(y) = \log_b(x) \log_a(y)$.
3. Find all solutions of the system of equations

$$x = \frac{1}{2} \quad y + \frac{1}{y}$$

$$y = \frac{1}{2} \quad z + \frac{1}{z}$$

$$z = \frac{1}{2} \quad t + \frac{1}{t}$$

$$t = \frac{1}{2} \quad x + \frac{1}{x}$$