

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 4, May 22, 2016

1. Find

$$S = \frac{1}{3} + \frac{1}{3^3} + \frac{1}{3^5} + \frac{1}{3^7} + \frac{1}{3^9} + \dots$$

Senior Questions

1. Prove that the equation $x^7 + y^9 = z^8$ has infinitely many solutions in positive integers $x; y$ and z , all powers of 2.
2. Every term of an infinite geometric progression is also a term of a given infinite arithmetic progression. Prove that the common ratio of the geometric progression is an integer.
3. The incircle of the quadrilateral $ABCD$ touches $AB; BC; CD$ and DA at $E; F; G$ and H respectively; see below
 - (a) Recall that the *incentre* of a triangle is the point where the internal angle bisectors of the triangle intersect. Show that the incentre of AEH lies on the incircle of $ABCD$.
 - (b) Show that the incentres of triangles HAE and FCG is perpendicular to the line joining the incentres of triangles EBF and GDH .

