

**MATHEMATICS ENRICHMENT CLUB.**  
**Problem Sheet 13, August 19, 2014<sup>1</sup>**

1. Let  $N = 1^9 \cdot 2^8 \cdot 3^7 \cdot 4^6 \cdot 5^5 \cdot 6^4 \cdot 7^3 \cdot 8^2 \cdot 9^1$ . How many perfect squares divide  $N$ ?
2. Let  $10 \leq a, b, c \leq 10$ . How many triplets,  $(a; b; c)$ , satisfy
 
$$\frac{a}{b} = \frac{a}{\frac{b}{c}}?$$
3. Find the sum of all primes  $p$  such that  $5^p + 4p^4$  is a perfect square.
4. Show that  $(1 + \sqrt[5]{5})^n + (1 - \sqrt[5]{5})^n$  is an even integer for all positive integers  $n$ .

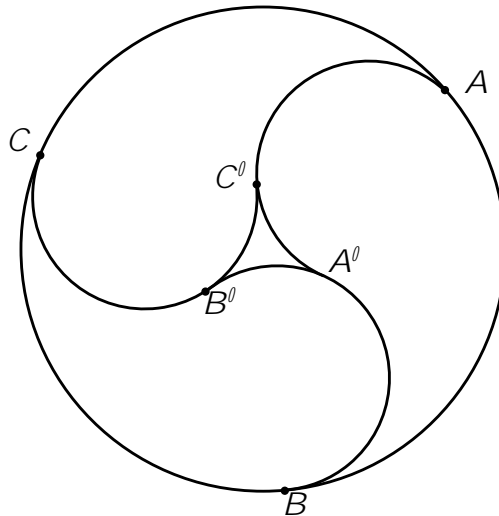


Figure 1: Figure for question 5

5. In the figure,  $ABC$  is a circle of radius  $R$  with 3 tear-drop shapes inside. Each of the arcs  $AC'A'$ ,  $BA'B'$  and  $CB'C'$  are of circles of the same radius,  $r$ . Find the ratio of  $R$  to  $r$  and the proportional area enclosed in the centre piece  $A'B'C'$ .
6. Arrange 11 points in the plane so that 16 lines can be drawn, each passing through 3 points.

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<sup>1</sup>Some problems from UNSW's publication *Parabola*

## Senior Questions

1. Show that, for  $x \in (-1, 1)$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$$

and hence show that

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

2. Using the above, how many terms are needed to approximate  $\ln(2)$  correctly to 5 decimal places?