$_n = 10 + (n \quad 1)d$. Now using the data $a_{a_2} = 100$ we get

$$100 = 10 + (a_2 \quad 1)d$$
$$90 = (a_2 \quad 1)d$$
$$90 = (10 + (2 \quad 1)d)d$$
$$0 = d^2 + 10d \quad 90; \quad \text{so}$$
$$d = \quad 9 \quad p$$

3. Note

$$\begin{aligned}125^{100} &= 5^{300}\\ &= \frac{10^{300}}{2^{300}}\\ &= \frac{10^{300}}{(1024)^{30}}\\ &= \frac{10^{300}}{(1.024 \times 10^3)^{30}}\\ &= \frac{10^{300}}{1.024^{30} \times 10^{90}}\\ &= \frac{10^{210}}{1.024^{30}}.\end{aligned}$$

It now remains to see that $1.024^{30} < 10$ and hence there are 210 digits. Can you show this using the binomial theorem?

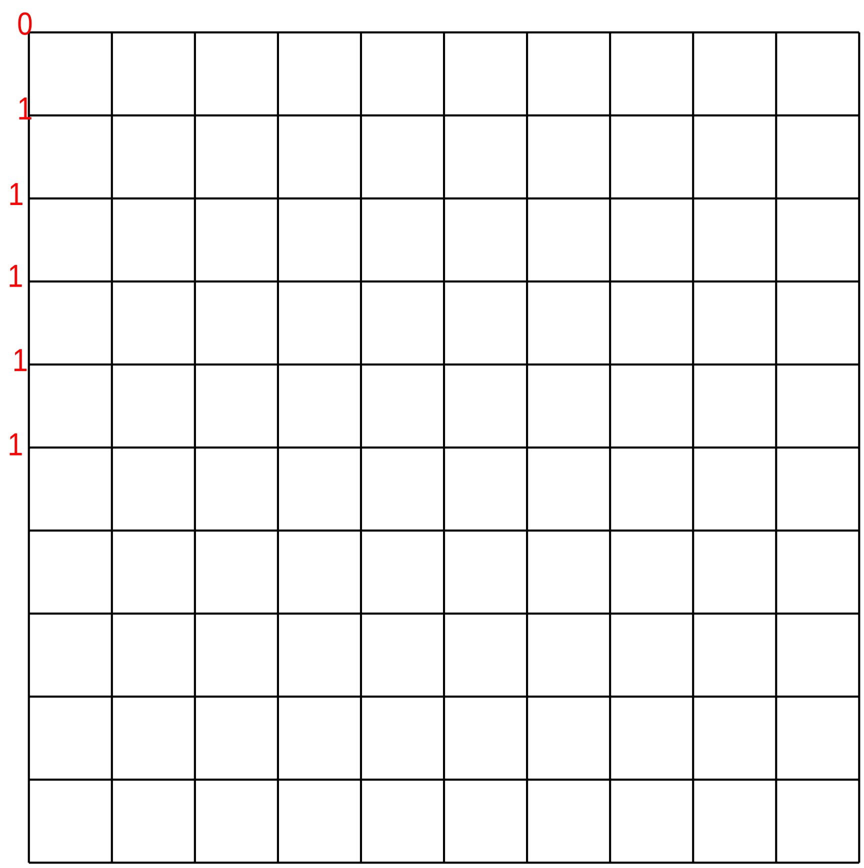
4. (a) See figure 1

Figure 1: The possible paths on a 2×2 grid. .

(b) At each vertex ask how many paths are there from the top-left to the given vertex. In fact, it is the sum of the number of paths from the top-left to the vertex immediately to the left of the given vertex and the number of paths from the top-left to the vertex immediately above the given vertex. This means the number of paths from the top-left to each vertex follows a Pascal's Triangle pattern. So there are $\binom{40}{20}$ paths.

(c) This generalises easily to $\binom{2n}{n}$.

5. Let's label the vertices of our hexagon 1 through to 6. Then we can refer to edges as (xy) where x and y are a vertex. Now since we can just re-label the hexagon however we want, let's just consider vertex 1. There are 5 edges coming off vertex 1, and since we only have 2 colours, 3 of these edges are the same colour, let's say red. And again, since we can re-label the hexagon however we want, let's suppose these edges are (12),(13) and (14). Suppose we don't have any red triangles, so (23) must be blue to prevent $\triangle$123 from being red, also (34) must be blue to prevent $\triangle$134 from being


0
1
1
1
1
1

all red and (24) must be blue to prevent 124 from being all red. But now (23), (34) and (24) are all blue, so 234 is blue.

Seeing as it doesn't matter how you label the hexagon, this always happens.

6. Look at the sequences in base-8, x_n always ends in either 3 or 5, while y_n always ends in 1. Hence they can never be the same.