MATHEMATICS ENRICHMENT CLUB.[1]

Solution Sheet 2, May 14, 2013

1. $LCM(10; 12) = 2^2 \quad 3 \quad 5 = 60$ minutes.

2. $6528(10a + 3) = 8256(30 + a)$ implies $a = 4$

3. 3 play only the piano.

4. Let $p(x) = (3 + 2x + x^2)^{1998} = a_0 + a_1 x + a_2 x^2 + \cdots$ $+ \quad (0) = 1998(3+2 \quad 0+0^2)^{1997} \quad (2+2 \quad 0) = 199$

   (b) $a_0 + a_1 + a_2 + \quad = p(1) = (3 + 2 + 1)^{1998} = 6^{1998}$

   (c) $a_0 \quad a_1 + a_2 \quad = p(\quad 1) = (3 \quad 2 + 1)^{1998} = 2^{1998}$.

5. (a) Since $a + b + c = 2$ and $a + b > c$, $a + c > b$ and $b + c > a$ each $a; b; c$

$$1 + ab + bc + ca \quad abc > 0$$

and

$$(a + b + c)^2 = 4$$
$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 4$$
$$ab + bc + ca = 2 \qquad 1$$

$$\frac{}{2}(a^2 + b^2 + c^2):$$

Combining the two yields the answer.

6. (a) Using the triangle inequality gives $AC < AB + BC$, $AC < AD + DC$, $BD < AD + AB$ and $BD < BC + CD$, summing all of these together gives

$$2(AC + BD) < 2(AB + BC + CD + AD)$$
$$AC + BD < p:$$

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.

Mark as E the intersection of AC and BD, then again using the triangle inequality we have $AB < AE + EB$, $BC < EB + EC$, $CD < CE + ED$ and $AD < ED + AE$. Again summing all of these together gives

$$AB + BC + CD + AD < AE + EC + BE + ED + BE + ED + CE + EA$$
$$p < AC + BD + BE + CA$$
$$p < 2(AC + BD)$$
$$\frac{1}{2}p < AC + BD:$$

(b) The lines AE, BE, CE and DE divide the quadrilateral into 4 pieces. Say $\backslash AEB =$, and $\backslash BEC =$, then by opposite angles $\backslash CED =$ and $\backslash AED =$. The 4 angles must sum to 2 so $2 + 2 = 2 \quad =) \quad =$. Note also that sin = sin( ). Now we may sum the area of the 4 triangles to determine the area of the quadrilateral:

$$a = \frac{1}{2}AE \quad BE \sin \; + \frac{1}{2}BE \quad CE \sin \; + \frac{1}{2}CE \quad DE \sin \; + \frac{1}{2}DE \quad AE \quad D \text{and} \; \backslash$$

A

O_1

O_2

P

C

B

O_3