

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 9, July 23, 2013

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1. The sequence $a_1; a_2; a_3; \dots$ is arithmetic. If $a_1 = 10$ and $a_{a_2} = 100$ what is $a_{a_{a_3}}$?
2. We play a game in which we try to get from one number to another. Each move we can replace the natural number m with ab if $a + b = m$ and a and b are both natural numbers. Can we get to 2001 from 22 in any number of moves?
3. How many digits does the number 125^{60} have?
4. Commander Keen is standing at the top left corner of an $n \times n$ grid, but wants to get to the bottom right corner. He's only allowed to move to the right, or downwards.
 - (a) Draw all the possible paths from the top left to the bottom right if the grid is 2×2 .
 - (b) How many possible paths are there if the grid is 20×20 ?
 - (c) What about $n \times n$?
5. Each of the six vertices of a regular hexagon are connected to every other vertex using either a red or a blue line. Show that, however this is done, the resulting diagram will always contain either a red or a blue triangle. Show that this is not always the case if we use the vertices of a pentagon.
6. Consider the two sequences $x_0 = 1; x_1 = 1; x_{n+1} = x_n + x_{n-1}$