MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 4, May 28, 2013

1. (a) Show that whatever base b is used, the number $(21)_b$ is never equal to twice $(12)_b$.

   (b) Find all the numbers and all bases b 12 for which there exists a two digit number $(ac)_b$ which is twice the number obtained by reversing its digits.

   (c) Find all bases b and all numbers $n = (ac)_b$ such that $n = 2$ $(ac)_b$.

2. In how many ways is it possible to write 1000 as a sum of consecutive odd integers?

3. Draw a right triangle ABC with right-angle at C and the sides marked $a; b; c$ as usual.

   (a) Draw the enlargement $A'B'C'$ of ABC by a factor of a.

   (b) On the same diagram draw the enlargement $A''B''C''$ $B'C'$ with $A''C''$, so that A', B' and new triangle $A'''B'''C'''$.

   (c) Explain why the angle at C''' is a right angle.

   (d)

Senior Questions

1. The hypotenuse of a right-angled triangle is 15 cm and the radius of the inscribed circle is 2cm. Find the perimeter of the triangle.

2. Suppose we place one of the numbers 1,2,3,...,2000 into each of 2000 boxes. Remove the two numbers a and b from any two boxes, chosen at random, and put their difference $a - b$ into one of the two boxes chosen and remove the empty box. Repeat the process until only one box remains. Show that the number in this box must be even.