

**MATHEMATICS ENRICHMENT CLUB.<sup>1</sup>**  
**Problem Sheet 1, May 7, 2013**

1. Laurie sold two cars for \$25 000 each. One he sold at a 20% profit and the other at a 20% loss. How much did he gain or lose ?
2. A number  $n$  has exactly 12 divisors. Given that  $n$  is divisible by 1,2,3,4,5 and 6 find a possible value of  $n$ . Is it the only one? Explain.
3. Without using a calculator, which is larger  $31^{24}$  or  $257^{15}$ ;
4. Let  $S_n = 2n(2n - 1)(2n - 2) \dots (n + 1)$ . For example,  $S_3 = 6 \cdot 5 \cdot 4 = 120$ .
  - (a) What is the power of 2 in the prime factorisation of  $S_n$  for  $n = 2;3;4 \dots$ ?
  - (b) Make a conjecture based on(i) and prove it.
5. Without using a calculator, show that

$$\sqrt[3]{\frac{9}{5} \sqrt{\frac{13}{18}}} - \sqrt[3]{\frac{9}{5} \sqrt{\frac{13}{18}}} = 3:$$

(Hint: Let  $x = a - b$  and cube.)

6. Let  $ABC$  be a triangle and  $D; E$  points on  $AB; BC$  respectively, and  $S$  be the intersection of  $AE$  and  $CD$ . If  $AD = DB$  and  $BE : EC = 2 : 1$ , find the ratios  $CS : SD$  and  $AS : SE$ :
7. (a) Let  $P$  be an interior point in an equilateral triangle  $ABC$ . Prove that we can always form a triangle with sides of length  $AP; BP; CP$ . (That is, we have to show that the sum of any two of these lengths is larger than the remaining one.)
  - (b) Give an example of a triangle and point inside it for which the above result is not true.

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<sup>1</sup>Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.