MATHEMATICS ENRICHMENT CLUB.[1]
Solution Sheet 1, May 7, 2012

Answers

1. Either $p|q$ or $p|q-1$ (1st condition), and either $q|p$ or $q|p+1$ (2nd condition). We have the following scenarios:

	$pm = q$	$pm = q-1$
$nq = p$	$p = q$	$mnq = q-1$ implies $p = 1 = q$
$nq = p+1$	$pmn = p+1$ implies $p = 1 = q$	$pm + nq = p + q$ implies $p = 1, q = 2$

2. Easy

3. Complete the square, then take di erence of two squares. Answers are $(x^2 - 2x + 2)(x^2 + 2x + 2)$ and $(x^2 - \sqrt{(2)}x + 1)(x^2 + \sqrt{(2)}x + 1)$.

4. Suppose $x \leq y \leq z$. Then $5/8 = 1/x + 1/y + 1/z \leq 3/x$, so $x < 5$. This means there are only 4 possible values for x.

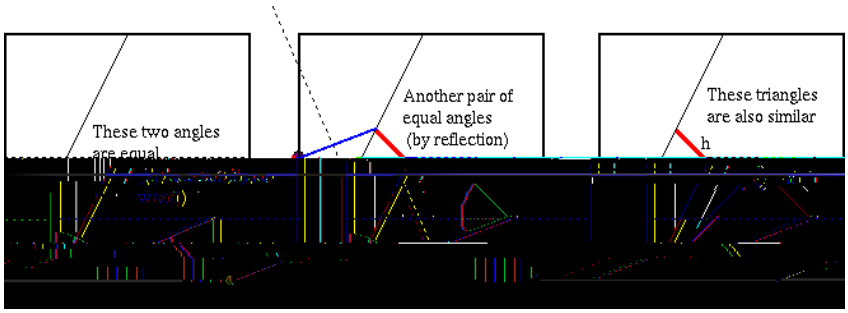
   $x = 1$ No solutions.

   $x = 2$ Solve $1/y + 1/z = 1/8$. So $8 \leq y \leq 2 * 8$. Testing y values in this range gives: $(9, 72), (10, 40)$.

   $x = 3$ Solve $1/y + 1/z = 7/24$. Since $1/4 < 1/y + 1/z < 1/3$. So $3 \leq y \leq 2 * 4$, Answers $(4, 24), (6, 8)$.

   $x = 4$ Solve $1/y + 1/z = 3/8$. Answers: $(3, 24), (4, 8)$.

8. You can check that the angles are the same for these triangles:



[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.