

MATHEMATICS ENRICHMENT CLUB.<sup>1</sup>  
Problem Sheet 7, June 9, 2012

1. In how many ways can 6 boys and 6 girls stand next to each other in a row such that no two boys stand next to each other and no two girls stand next to each other?
2. The notation  $5!$  means  $5 \times 4 \times 3 \times 2 \times 1 (= 120)$ . How many zeros are there at the end of  $1000!$ .
3. (a)  $a; b$  are positive numbers with  $a + b = k$ . Explain why  $ab$  is greatest when  $a = b = \frac{k}{2}$ .  
(b) Suppose that  $x^2 + y^2 = c^2$ , find the minimum value of  $x^4 + y^4$ .
4. (a) Show that there are infinitely many non-zero integers  $x; y; z$  such that  $2^x + 2^y = 2^z$ .  
(b) Show that if  $n > 2$  then there are no nonzero integers  $x; y; z$  such that  $n^x + n^y = n^z$ .
5. (Parts b and c require Year 9 and Year 10 Mathematics).  
Let  $ABC$  be an isosceles triangle with the base angles  $B$  and  $C$  being  $72^\circ$  and  $AB = AC = 4$ . The length of the base  $BC$ , called  $x$  is chosen such that a line  $CD$  can be drawn, where  $D$  lies on  $AB$ , such that  $\angle BDC = 72^\circ$ .
  - (a) Find a pair of similar triangles and show that  $x$  satisfies,  $x^2 + 4x - 16 = 0$ .
  - (b) Use triangle  $ABC$  to find  $\cos 72^\circ$  in surd form.
  - (c) Use triangle  $ACD$  to find  $\cos 36^\circ$  in surd form.
6. Suppose that two non-parallel straight lines  $k$  and  $l$  meet at a point  $P$  which is **not** on the page of my book. Construct a line which would (if  $P$  did lie on the page) bisect the angle between the lines and pass through  $P$ .
7. Let  $K; L$  be points on the sides  $AB; AD$  respectively of the convex quadrilateral  $ABCD$  such that  $AK = \frac{1}{3}AB$  and  $AL = \frac{1}{3}AD$ . Similarly,  $M; N$  are points on  $CD; CB$  such that  $CM = \frac{1}{3}CD$  and  $CN = \frac{1}{3}CB$ .

## Year 11 Question.

1. Suppose that  $m$  and  $n$  are positive real numbers. Use trigonometry to find the maximum value of

$$\frac{m+n}{\sqrt{m^2+n^2}};$$