

MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 3, May 14, 2012

1. The perimeter of a base of a rectangular brick with integer sides is 18 cm, whilst its volume is 42 cm³. What is its height?

2. Calculate

$$1 - \frac{1}{2} \quad 1 - \frac{1}{3} \quad 1 - \frac{1}{4} \quad \dots \quad 1 - \frac{1}{2008} \quad ;$$

3. Find the smallest positive integer whose square ends in (a) 09 and (b) 9009.

4. Show that if a, b are positive numbers such that $ab \leq 1$ then

$$\frac{a}{b+1} + \frac{b}{a+1} + (1-a)(1-b) \leq 1;$$

5. Suppose we have the numbers $x_0 = 0; x_1 = 1$ and $x_{n+1} = x_n + 2x_{n-1}$ for $n \geq 2$.

a. Write down the numbers x_n for $n = 2; 3; 4; 5; 6$.

b. Show that there is no n for which x_n

Senior Questions.

1. Find $\int_0^1 \frac{1}{1+t+t^2}$.

2. Find the limit $\lim_{n \rightarrow \infty} \frac{1}{n!}$.