

7ci fgY⁻Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
DYM'f'G`YnU_	d"g`YnU_4 i bgk "YXi "Ui	•	F cca '' %) ž A cfj Yb 6fck b	-',)!&(&&

GWXcc``7cbHJWhʻ±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b['<ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . 'Ž * %'&'-' ,) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y'7ci fgY

H\Y'5 ffg/ 'GcVJU\`GVJYbW\g'7 UdglcbY']g'U'a i `h]!X]gVJd`]bUfm\wci fgY'h\Uhdfcj]XYg'mci 'k]h\ 'Ub'cddcfhi b]hm hc'fYZYWhcb'mci f'X]gVJd`]bUfmghi X]Yg'cj Yf'h\Y'dfYj]ci g'h\fYY'`Yj Y`g'cZghi Xm'' \delta'dUfh]W\ 'Ufz'h\Y'Wci fgY'k]`` Wcbg]XYf'WUgg]Wg']b'h\Y'`]hYfUhi fY'cZ`]VYfU`'YXi WUh]cb'hc'Ug_'hc'k \UhYI hYbhmci f'ghi Xm\Ug'`]j YX'i d'hc'h\Y j Uf]ci g'WcbW\dh]cbg'UbX']XYU'g'cZU'i b]j Yfg]lmYXi WUh]cb'''Mci '\Uj Y'U'W\UbW\'hc'h\]b_'UVci hmci f'ck b X]gWJd']bY']b'Wcbj YfgUh]cb'k]h\ 'ghi XYbhg'Zrca 'ch\Yf'X]gWJd']bYgz'k \]'Y'XYj Y`cd]b['mci f'ck b'Uk UfYbYgg'cZ h\Y'X]gWJd']bUfm_bck 'YX[Y'UbX'i bXYfgHUbX]b['mci '\Uj Y'UWei]fYX"

H\Y`WcifgY`]g`U]a YX`UhUgg]gh]b[`mci`k]h\`h\Y`hfUbg]h]cb`Zfca`mcif`W`ffYbhXY[fYY`hc`mcif`fc`Y`]b`h\Y`kcfXUg`W]h]nYbg`]b`Yad`cmaYbhcf`Zfh\Yf`YXiWUh]cb"

BchY. 'H\Y'Wci fgY'k]```VY'a UXY'Uj U]`UV`Y'Zcf'Gi a a Yf'gYgg]cb'dfcj]X]b['a cfY'ZYI]V]`]mmZcf'ghi XYbhg a YYh]b['h\]g'fYei]fYa YbhcZh\Y]f'dfc[fUa #a U'cf'ghfYUa "': cf'ghi XYbhg'Zcf'k \ca 'gi a a Yf']g'bchimci f'`Ugh gYgg]cb'cZYbfc`a Ybhz'mci 'a i ghcVhU]b'Uddfcj U`'Zfca 'mci f'a U'cf'Wcbj Ybcf'hc Ybfc`']b'h\Y'WUdghcbY dfcj]XYX'mci '\Uj Y'Wca d'YhYX'h\Y'bYWfggUfmWci fgYg']b'mci f'a U'cf'ghfYUa "'9bfc`a Ybh]b'h\Y'WUdghcbY fYei]fYg'ZJW 'hmUddfcj U`"D`YUgY'WcbhUWh\Y'<u>. UW 'hmGhi XYbh7 YbhfY</u>"

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 5fn]W `UhY`Ub`i bXYfghUbX]b[`cZh\Y`h\YcfYh]WU``UbX`a Yh\cXc`c[]WU``df]bV\]d`Yg`cZh\Y]f`X]gW]d`]bY &" 8Ya cbghfUhY`Wca dfY\Ybg]cb`cZYh\]WU``UbX`gcW]U``fYgdcbg]V]`]h]Yg`]b`h\Y`dfUWf]WY`cZh\Y]f`X]gW]d`]bY \" 5fn]W `UhY`h\Y`k Umg`]b`k\]W\`h\Y]f`X]gW]d`]bUfmhfU]b]b[`Wci `X`VY`i gYX`]b`Z fh\Yf`ghi Xmcf`h\Y`k cfX cZk cf
- (" 7 f]h]wU``mYj U`i UhY`UbX`fYZYWicb`_YmUf[i a Ybhg`UbX`XYVUhYg`]b`h\Y`\i a Ub]h]Yg`UbX`gcW]U` qW]YbWYg"

HYUWX]b[GhfUhY[]Yg

H\Y'5ffg'/ 'GcWJU''GWJYbWfg'7 UdghcbY'dfcj]XYg'gh XYbhg'k]h\'h\Y'cddcfh b]hmhc'fYZYWhcb'h\Y]f X]gWJd']bUfmgh X]Yg'cj Yf'h\Y'dfYj]ci g'h\fYY''Yj Y`g'cZgh Xm''5g'k Y``'Ug'XYa cbghfUhJb['h\Y]f'i bXYfghUbX]b[cZh\Y'a U'cf'h\YcfYhJWU''UbX'a Yh\cXc`c[]WU''df]bWJd`Yg'k]h\]b'h\Y]f'X]gWJd`]bYZgh XYbhg'k]``'Wcbg]XYf'h\Y Yh\]WU'`UbX'gcWJU''fYgdcbg]V]`]hJYg'UggcWJUhYX'k]h\'h\Y]f'UfYU'cZgh XmUbX'k]``'cc_'hc'h\Y'Z h fYhc i bXYfqhUbX'\ck 'h\Y]f'qh X|Yg'\Uj Y'dfYdUfYX'h\Ya 'Zcf'U'WUfYYf'cf'Z fh\Yf'qh Xm'

H\Y`Wci fgY`k]```VY`ghi XYbhiWYbhfYX`UbX`gYa]bUf`Xf]j Yb"`H\Y`a U^cf`UggYgga YbhiHug_`k]```VY`U`fYgYUfW\YggUmk\]W\k]```U``ck `ghi XYbhg`hc`XYa cbghfUhY`h\Y]f`UW\]Yj Ya YbhcZh\Y; fUXi UhY`5hff]Vi hYg`Zcf`h\Y]f`UfYU`cZghi Xm'`5`^ci fbU``dcfhZc`]c`k]```VY`_Ydhi\fci [\`h\Y`Wci fgY`k\]W\`k]```U``ck `ghi XYbhg`hc`fYZYWhcb h\Y]f``YUfb]b[`cj Yf`h\Y`h\fYY`Yj Y`g`cZh\Y]f`a U^cf"

 $\label{eq:hampyweigh} $$H\Y'WcifgY'k_j^{-}]bjc_jY'gYa_bUf!gfmY']bhYfUWfjY'X]gW'gg]cb'VUgYX'cb'h\Y']XYUg'UbX'gh]a_i`i_g'cZk_YY_m fYUX]b[_g''DUfh]WJoUbhg'k_]^{-}VY'fYei_]fYX'hc'a_U_Y'hk_c'dfYgYbhUh]cbgž'YUX]b[_'X]gW'gg]cb'Vm[_]j_]b[_'U'Wf]h]WJ'UbU'mg]g'UbX'cj_Yfj_]Yk_'cZ_Ym]XYUg']b'h\Y'fYUX]b[_g'Ug'h\YgY'VYUf'cb'h\Y]f'ck_b'X]gWJd']bY'UbX'j_]Yk_g'' $$7f]h]WJ''ZUbU'mh]WJ''h\]b_]b[_'k_]^{-}U'gc'VY'XYj_Y'cdYX'h\fci_[_\'Ub'YI_hYbXYXZ'Wf]h]WJ''UWJXYa_]WYggUmcb'U hcd]Whc'VY'WkcgYb']b'Wcbgi_'hUh]cb'k_]h\'n\Y'YWi_fYf''$

fYdcfhg"

5ggYgga Ybh' . A]X!hYfa 'HU_Y!\ca Y 'Ugg][ba Ybh

GHJfh:XUhY.

@Yb[h\.'%) \$\$ 'k cfXg

 $8 YHJ]^*g. \% $$^k cfXg'''Gh XYbhg'a i ghWf]hJWJ^*mUbU^mgY'Ugg][bYX'UfhJWYg''': YYXVUW_j JU']bX]j]Xi U^*Wcaa Ybhg''$

5XX]h]cbU`XYhU]`g.

5ff]WYg'Zcf'Wf]h]WU`'UbU`mg]g'k]``'VY'a UXY'Uj U]`UV`Y'cb'A ccX`Y"

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$

5ggYgga Ybh (. FYZYWhjj Y Ugg][ba Ybh

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\., \$\$ k cfXg

8 YHJ]`g. `, \$\$`k cfXg"`Gh XYbhg`a i ghUfh]W``UhY`h\Y`k Umg`]b`k \]W\`h\Y]f`X]gW]d`]bUfmhfU]b]b[`'Wci `X`VY`i gYX]b`Z fh\Yf`gh Xmcf`h\Y`k cfX`cZk cf_"

5XX]h]cbU`XYhU]`g.

Hc \Y`d`a U_Y`h\Y`Ugg][ba Ybha cfY`ZcW ggYXžgca Y`Uff]WYg`k]``VY`dfcj]XYX`cb`A ccX`Y`k \]W\`a UmVYfYZYffYX`hc"

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM & gYM & gYY & fb & gYY &$

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

7 ci fgY fYUX]b[g k]``VY Uj U]`UV`Y cb`]bY]b A ccX`Y"

FYWca a YbXYX FYgci fWYg

Bch'Uj U]`UV`Y

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Gh XYbhZYXVUW_k]``VY`h\fci [\`h\Y`Ubcbma ci g`cb`]bY`A m'91 dYf]YbWY`gmghYa "

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUlfYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W±bZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

- fYei YghgʻZcfʻYl hYbg]cb/`UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ UbX GUZYmdc`]W]Yg/
- YI Ua]bUh]cb dfcWXi fYg/